



IIT ROORKEE



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CERTIFICATION COURSE

Lecture 6: Basic Optics for Optical Sensing-IV

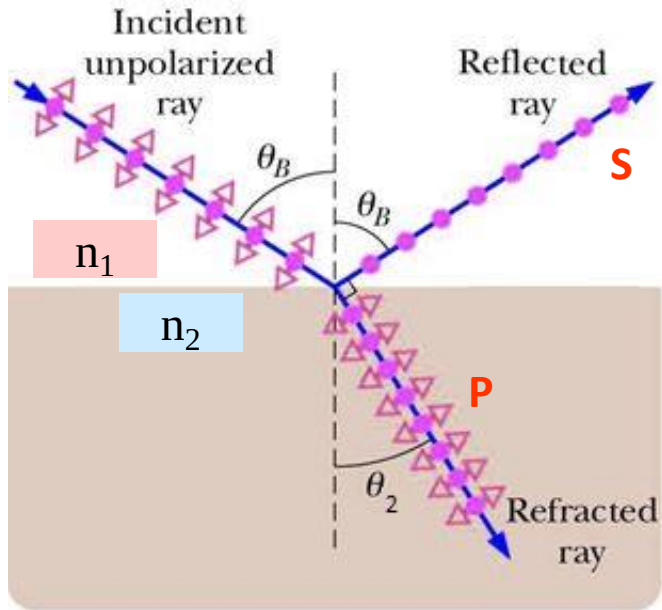
Polarization by reflection – Brewster angle sensor, Total Internal Reflection

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Polarization by Reflection: Brewster Angle



- Component perpendicular to page
- ↔ Component parallel to page

$r_p = 0$ Together with Snell's law

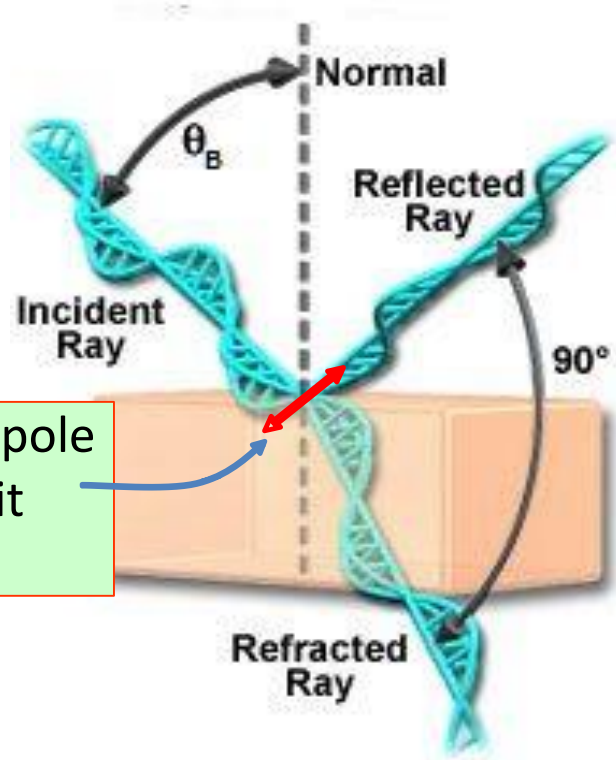
$$r_p = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$\tan \theta_B = \frac{n_2}{n_1}$$

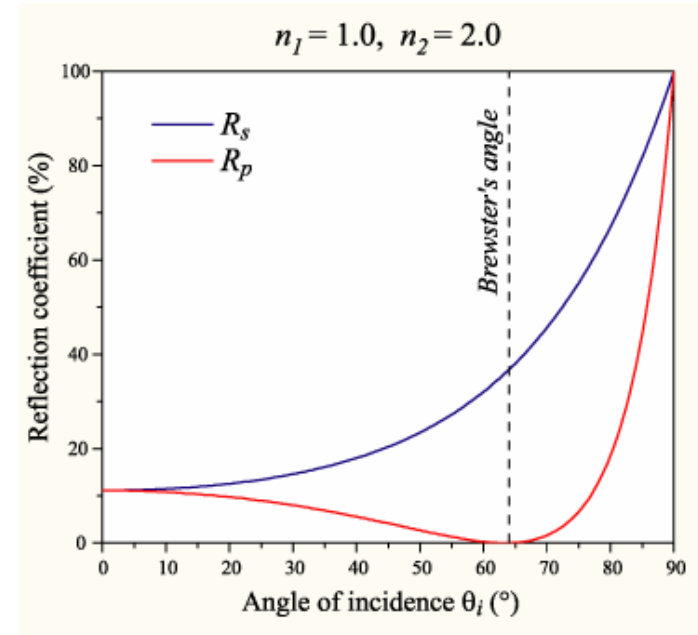
$$\theta_B + \theta_2 = 90^\circ$$

$$n_1 \sin \theta_B = n_2 \sin \theta_2 = n_2 \cos \theta_B$$

Brewster Angle – Physics



Example

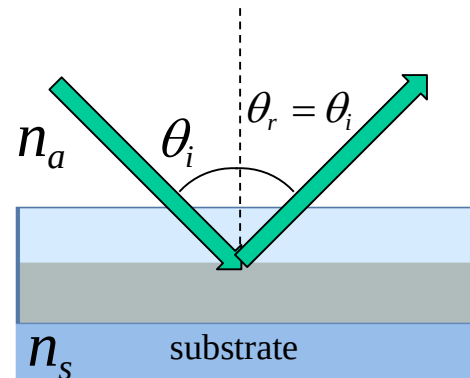


Can we do Sensing using Brewster Angle?

$$\tan \theta_B = \frac{n_s}{n_a}$$

Use: $\frac{d \tan x}{dx} = \frac{1}{\cos^2 x}$ To get:

$$\frac{d \tan \theta_B}{dn_a} \frac{1}{\cos^2 \theta_B} \frac{d \theta_B}{dn_a} = -\frac{n_s}{n_a^2} \Rightarrow \frac{d \theta}{dn_a} = -\cos^2 \theta_B \frac{n_s}{n_a^2}$$



Sensitivity

$$S = \frac{d \theta_B}{dn_a} = -\frac{1}{1 + \tan^2 \theta_B} \frac{n_s}{n_a^2} = -\frac{1}{1 + n_s^2 / n_a^2} \frac{n_s}{n_a^2} = -\frac{n_s}{n_a^2 + n_s^2}$$

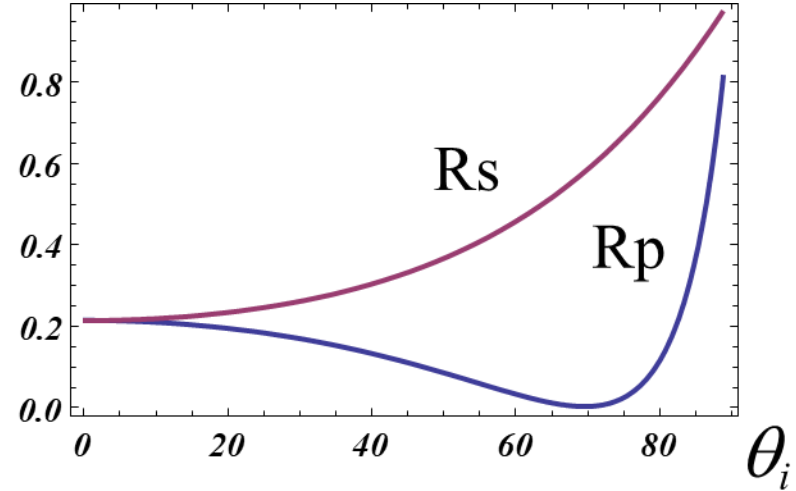
Example: water on Si substrate:

$$S = -\frac{3.6}{1.33^2 + 3.6^2} \quad \text{Relatively small sensitivity!}$$

$$\approx 0.244 \text{ rad} / \text{RIU} = 14 \text{ deg} s / \text{RIU}$$

Another problem is the large width of the curve!

Low FOM



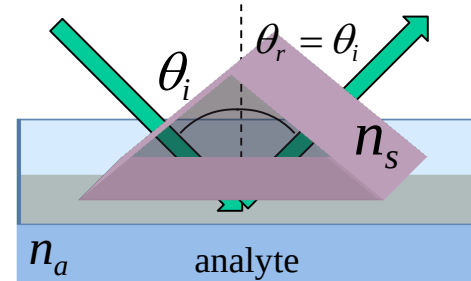
Can we do Sensing using Brewster Angle? Incidence from the Substrate Side (prism)

$$\tan \theta_B = \frac{n_a}{n_s}$$

Use: $\frac{d \tan x}{dx} = \frac{1}{\cos^2 x}$ To get:

$$\frac{d \tan \theta_B}{dn_a} \frac{1}{\cos^2 \theta_B} \frac{d \theta_B}{dn_a} = \frac{1}{n_s} \Rightarrow \frac{d \theta_B}{dn_a} = \cos^2 \theta_B \frac{1}{n_s}$$

$$S = \frac{d \theta_B}{dn_a} = - \frac{1}{1 + \tan^2 \theta_B} \frac{1}{n_s} = \frac{1}{1 + n_a^2 / n_s^2} \frac{1}{n_s} = \frac{n_s}{n_a^2 + n_s^2}$$



The same expression as before!

Bio-World and Brewster Angle

Brewster angle for the air/water interface is about 53deg., at which maximum polarization in reflection occurs !

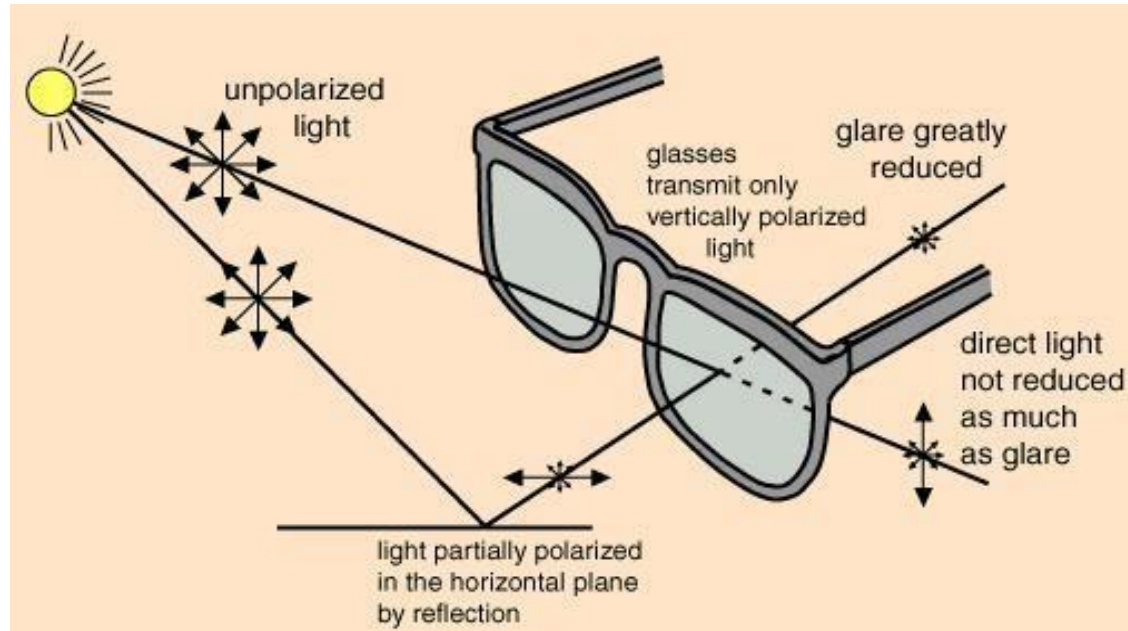


R. Wehner, J. Experimental Biology, 204, 2589, 2001 Landscape by Sabkhat al Muh, south-east of Tadmur, Syria

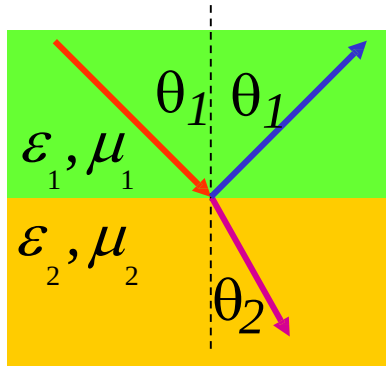


Water beetles, bugs and birds use this fact to detect bodies near the water surface – **depends on the sun's height !**

Cutting down glare from reflections at Brewster



Total Internal Reflection-TIR

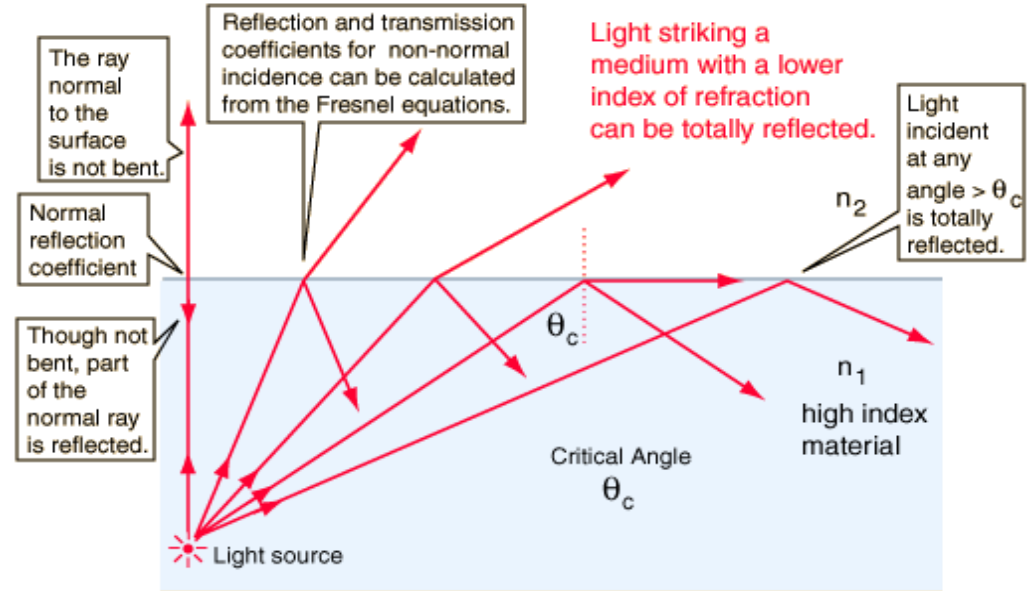


Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

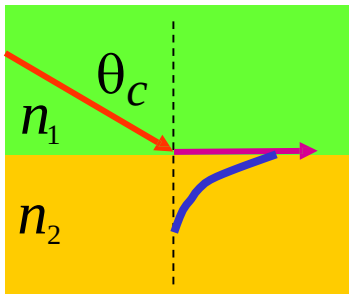
When $n_1 > n_2$ then at certain incidence angle we get: $\theta_2 = 90^\circ$

The critical angle: $\sin \theta_c = \frac{n_2}{n_1}$



For $\theta_1 > \theta_c$ the refraction angle is imaginary

$$\cos \theta_2 = \sqrt{\left[\frac{n_1 \sin \theta_1}{n_2} \right]^2 - 1}$$



We get a decaying wave across the surface

Evanescent Wave!

$$E \propto \exp(-z / d)$$

Penetration Depth

$$d = \lambda_0 / (2\pi \sqrt{n_1^2 \sin^2 \theta_1 - n_2^2})$$

Fresnel Equations at Total Internal Reflection

Using Fresnel formulae we get:

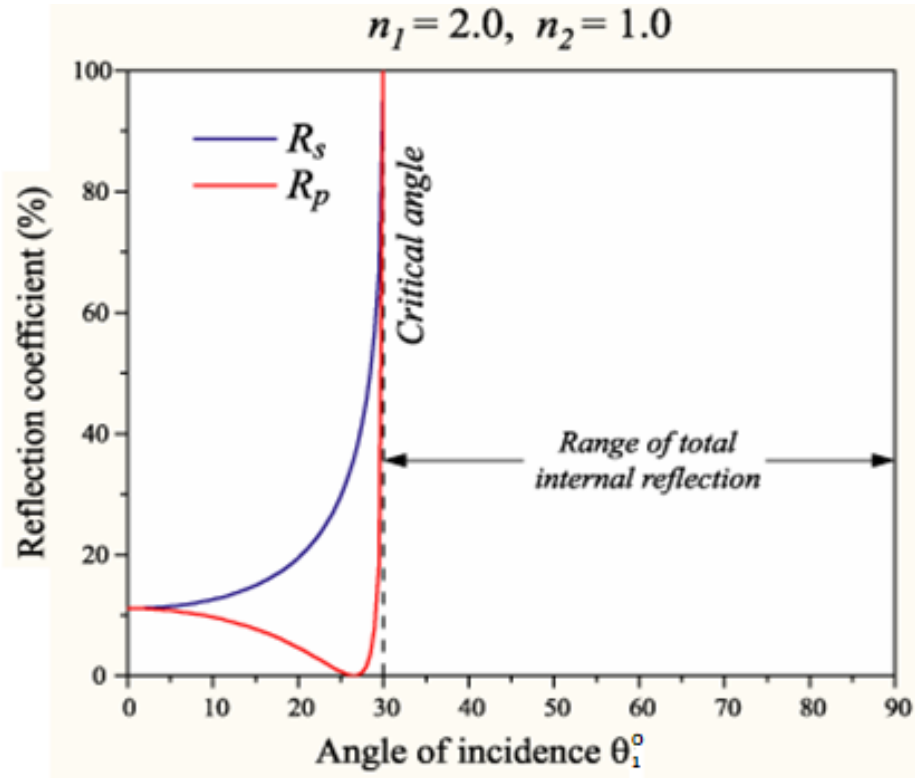
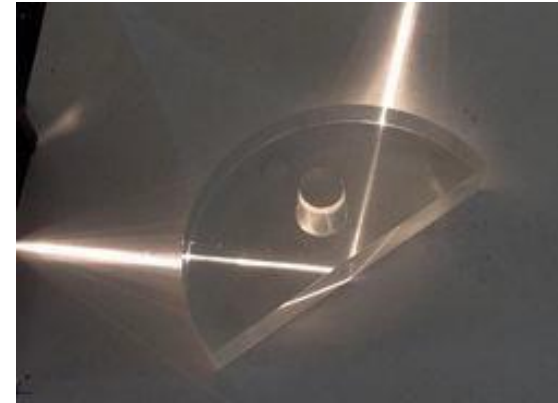
$$r_p = \frac{n_{12}^2 \cos \theta_1 - i\sqrt{\sin^2 \theta_1 - n_{12}^2}}{n_{12}^2 \cos \theta_1 + i\sqrt{\sin^2 \theta_1 - n_{12}^2}}$$

$$r_s = \frac{\cos \theta_1 - i\sqrt{\sin^2 \theta_1 - n_{12}^2}}{\cos \theta_1 + i\sqrt{\sin^2 \theta_1 - n_{12}^2}}$$

The phase:

$$\left. \begin{aligned} \tan \frac{\delta_p}{2} &= -\frac{\sqrt{\sin^2 \theta_1 - n_{12}^2}}{n_{12}^2 \cos \theta_1} \\ \tan \frac{\delta_s}{2} &= -\frac{\sqrt{\sin^2 \theta_1 - n_{12}^2}}{\cos \theta_1} \end{aligned} \right\} \tan \frac{\delta_p - \delta_s}{2} = \frac{\cos \theta_1 \sqrt{\sin^2 \theta_1 - n_{12}^2}}{\sin^2 \theta_1}$$

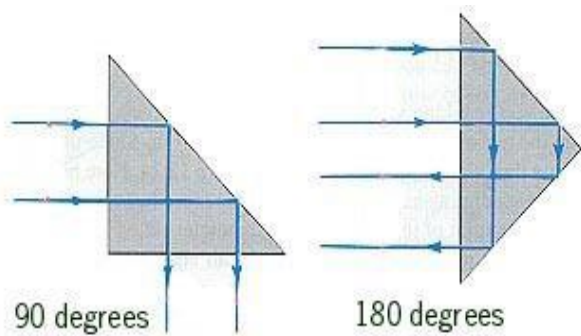
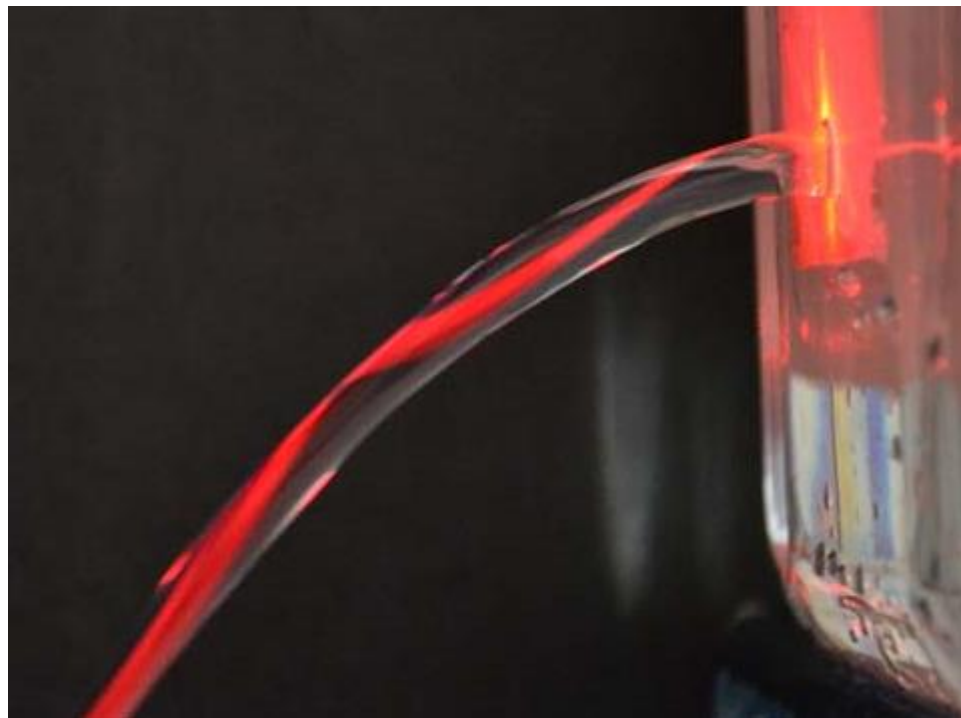
TIR - Example



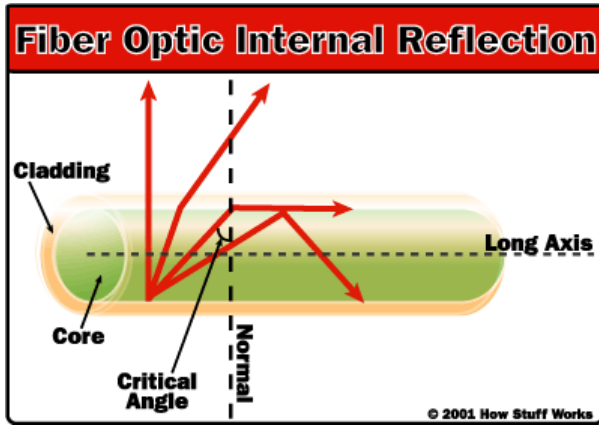
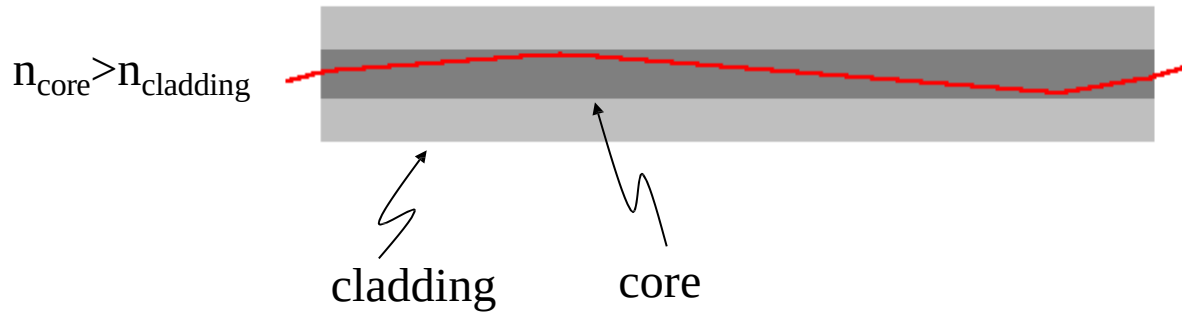
TIR or ATR

In water streams

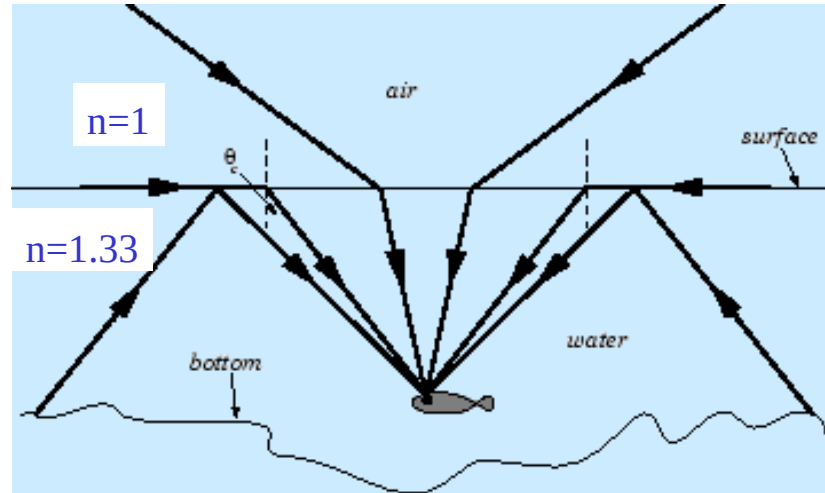
Prisms as Mirrors



TIR in Optical Fibers

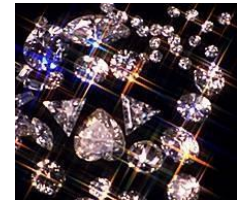


TIR, Fish Vision and Diamonds Brilliance



The fish sees the sky if it looks upwards but seem the bottom of the pond if it looks at angle larger than 48.8 degs.

Question: explain the brilliance of diamond in air knowing that for diamond $n=2.42$ and explain why it is less shiny when it is in water?



Summary

1. Polarization by reflection at Brewster angle and underlying Physics was discussed.
2. The potential use and limitations of Brewster angle for sensing applications was presented.
3. Total internal reflection for angles $\theta_1 > \theta_c$ was discussed and the expression for penetration depth of evanescent waves was obtained.

Thank You